$$CD:AC=BC:AB, \text{ or } CD:a=b:c,$$

from which $CD=ab/c$, which agrees with result (2).

Also solved by *J. R. HITT*, *J. SCHEFFER*, and *G. B. M. ZERR*.

CALCULUS.

144. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Find volume common to the two solids

$$[x/a]^{\frac{2}{3}}+[y/b]^{\frac{2}{3}}=[z/c]^{\frac{2}{3}}, \quad [y/b]^{\frac{2}{3}}+[z/c]^{\frac{2}{3}}=[x/a]^{\frac{1}{3}}.$$

Solution by the PROPOSER.

From the equations $[x/a]^{\frac{2}{3}}+[y/b]^{\frac{2}{3}}=[z/c]^{\frac{2}{3}}$ and $[y/b]^{\frac{2}{3}}+[z/c]^{\frac{2}{3}}=[x/a)^{\frac{1}{3}}$ we find the limits of z to be $z=c\{[x/a]^{\frac{2}{3}}+[y/b]^{\frac{2}{3}}\}^{\frac{3}{2}}$ to $z=c\{[x/a]^{\frac{1}{3}}-[y/b]^{\frac{2}{3}}\}^{\frac{3}{2}}$. Eliminating z we get

$$y=\pm\frac{b}{\sqrt{8}}\left[\frac{x}{a}\right]^{\frac{1}{2}}\{1-[x/a]^{\frac{1}{3}}\}^{\frac{3}{2}}=y'.$$

The limits of x are $x=0$ to $x=a$.

$$\therefore\ V=2c\int_0^a\int_0^{y'}[\{(x/a)^{\frac{1}{3}}-(y/b)^{\frac{2}{3}}\}^{\frac{3}{2}}-\{(x/a)^{\frac{2}{3}}+(y/b)^{\frac{2}{3}}\}^{\frac{3}{2}}]dxdy.$$

Let $x/a=u^3$, $y/b=v^3$.

$$\therefore\ V=18abc\int_0^1\int_0^{\sqrt{[\frac{1}{2}u(1-u)]}}u^2v^2\{(u-v^2)^{\frac{3}{2}}-(u^2+v^2)^{\frac{3}{2}}\}dudv$$

$$=\tfrac{3}{8}abc\int_0^1\left(u^2(2+u+2u^2)\sqrt{(1-u^2)}-8(1-u^2)^{\frac{5}{2}}+3u^2\sin^{-1}\left[\frac{1-u}{2}\right]\right.$$

$$\left.+6u^5\log\left[\frac{1+\sqrt{(1-u^2)}}{u}\right]\right)u^2du$$

Let $u=\cos\theta$.

$$\therefore\ V=\tfrac{3}{16}abc\int_0^{\frac{1}{2}\pi}\left(4\cos^2\theta\sin\theta+2\cos^3\theta\sin\theta+4\cos^4\theta\sin\theta-16\sin^5\theta+3\theta\cos^2\theta\right.$$

$$\left.+12\cos^5\theta\log\left[\frac{1+\sin\theta}{\cos\theta}\right]\right)\cos^3\theta\sin\theta d\theta=\frac{115\pi abc}{2048}.$$